NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC²ML
UNIT
BRIEF

NC Math 1 – Linear Functions

Function is a conceptual category within the NC High School Mathematics Standards. Prior to revisions, confusion existed around which function families were addressed within each course and some standards appeared in multiple courses. The standards, the Mathematics Resources for Instruction, and the Revisions documents provide clarity and outline the progression of the study of function families across the NC Math 1-3 courses**.**

NC Math 1: Linear, Quadratic, & Exponential
NC Math 2: Quadratic, Square Root, & Inverse Variation
NC Math 3: Exponential, Logarithmic, Polynomial, Rational, Absolute Value, & Trigonometric

The *Function* concept is a vital mathematical concept that is distinguished from "expression", "equation", "formula" and other mathematical relations. A relation that is a *function* must be

> *A relation that pairs every element in one set, called the domain, with exactly one element of a second set, called the range.*

Always relating an input element to its only output element is the predictive, pattern-like power of functions that make them so useful within mathematics and beyond.

ALGEBRA AND FUNCTION ACROSS K-8

Algebra and *Function* are not separate domains in K-5, as students begin to express patterns and generalizations with variables and graph ordered pairs. Students use variables to express mathematical relationships, and progress toward utilizing equations in which variables represent quantities that vary in relation to one another.

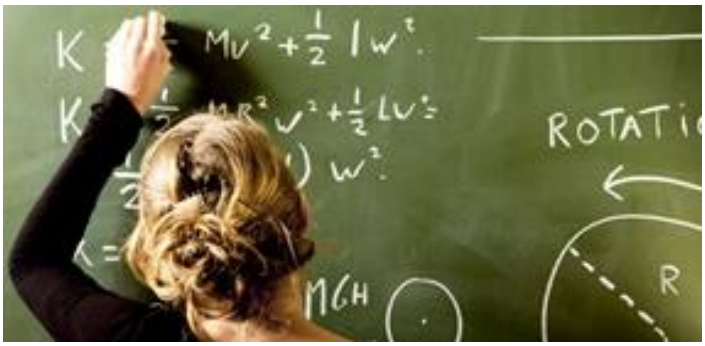
In grades 6 and 7 pattern generalization continues as students investigate ratios and proportions, and grade 8 is when students are introduced to a definition of function. In 8th grade, students experience how functions describe relationships between quantities using words, variables, and graphs; and particular attention is given to linear relationships and the change in one quantity with respect to another quantity.

The use of variables to describe functional relationships keeps *Algebra* and *Function* connected throughout grades 9-12. Algebra can be related to Function as a tool for better understanding characteristics of functional relationships, when those relationships are expressed symbolically. 8th grade students focus primarily on linear functions, their graphical representations, and their graphical characteristics highlighted within the form $y = mx+b$.

Function notation is not explicit in the standards until NC Math 1. Though the notation is familiar to teachers and ubiquitous across mathematics curricula, "y = f(x)" is a single mathematical sentence that packs a lot of meaning. This sentence shows that "y" and "f(x)" are the same value. It names a function "f", with a generalized input variable "x", and a function value or output "f(x)".

NC MATH 1 - LINEAR FUNCTIONS UNIT

The second unit of the Collaborative Pacing Guide for NC Math 1 is the *Linear Functions* unit. Keeping the connection between *Algebra* and *Function*, in this unit students are expected to recognize the graphical meaning of parameters ***a*** and ***b*** within the equation ***f(x) = ax + b*** (F-LE.5) and be able to manipulate the variables, justifying their algebraic methods of solution (A-REI.1). Not only will students generate linear equations to model situations (A-CED.1, 2), identifying the meaning of the coefficients and variables



within the context(A-SSE.1), but they will also understand that the graph of the equation shows all possible domain and range pairs (F-IF.5) that are solutions to the equation (A-REI.10).

Sequences are functions with domains that are very much like the set of natural numbers. In this unit students will build explicit and recursive arithmetic sequence (linear) expressions from descriptions, graphs, and tables of values (F-BF.1a).

This unit also includes the applications of linear relationships to geometry and statistics. Students will understand linear relationships that are represented in the coordinate plane or by formulas to investigate the geometric characteristics of parallel, perpendicular, midpoint, perimeter, and area (G-GPE.5). Statistical applications of linear relationships include fitting a least squares regression line to a scatterplot, assessing the goodness of fit with the correlation coefficient and residual plots, and then using that relationship as a model for prediction (S-ID.6a,b).

RATE OF CHANGE & COVARIATION IN NC MATH 1

For some functions, patterns of behavior in their output categorize them into particular function families. Building from students' prior understanding of rate, students in NC Math 1 are first exposed to linear functions – functions that have a constant *additive* rate of change. That is, over equal intervals of input, the output changes by the same amount. Following this, students engage with quadratic and exponential functions as families of functions that do not have a constant additive rate of change.

Exponential functions have what is called a *multiplicative* rate of change. That is, over equal intervals of input, the output of the function changes by a constant factor, thus the rate of change of the function is proportional to the value of the function. Quadratic functions lack constant *additive* or *multiplicative* rates of change.

To understand the behavior of a function and its' function family, teachers often support students by developing a rule of correspondence between input and output by focusing on the action of plugging in numbers and analzying the output. This approach is called a *correspondence* perspective of function. While useful, research has shown that by also attending to a *covariation* perspective of function, students can develop a stronger and dynamic understanding of rate of change across function families (Oehrtman et al., 2008). A covariation view of function focuses on coordinating the change in two varying input and output values as one moves through a table or graph. This view makes the concept of rate of change explicit by highlighting repeatability in the input and output. Instruction and tasks that draw on this perspective engender student's prior understanding of rate and supports them in making sense of the uniqueness of individual function families. In addition, this approach has been found to support conceptual understanding of inverse functions, function compositions, and calculus, which students engage with in NC Math 2, NC Math 3, and undergraduate mathematics.

QUESTIONS TO CONSIDER

- *What conceptions do your students commonly have about linear functions?*
- *How could a covariational perspective support your students?*
- *How can you design appropriate interventions or tasks to advance or refine their conceptions?*
- *Given that sequences are functions, what are some limitations to the vertical line test?*

References

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, 352-378.

LEARN MORE

Join us as we journey together to support teachers and leaders in implementing mathematics instruction that meets needs of North Carolina students.

NC²ML MATHEMATICS ONLINE

For more information and resources please visit the NC DPI math wiki for instructions on accessing our Canvas page created in partnership with the North Carolina Department of Public Instruction by http://maccss.ncdpi.wikispaces.net/

North Carolina Collaborative for Mathematics Learning

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