NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC²ML UNIT BRIEF

NC Math 2 - Transformations

Transformation content appears in both the NC Middle School Mathematics Standards and NC High School Mathematics Standards. Students begin learning about location and representations within the coordinate plane in 5th grade and are introduced to translations, rotations, and reflections in 8th. The Mathematics Resources for Instruction highlights connections between 8th grade and NC Math 2 and the pre-requisite experiences for understanding transformations as functions that can verify congruency or similarity.

Transformations are the focus of the first unit of NC Math 2, within the NC Collaborative Pacing Guide. Traditionally, transformations were seen as actions applied to geometric objects without connections to functions, beyond the fact that their pre-images and images may be drawn within the coordinate plane. It also may have then been noted that rigid transformations produce images that are congruent to their pre-image and dilations create images that are similar, but not congruent (unless you applied an uninteresting scale factor of 1).

NC MATH 2 – UNIT 1, TRANSFORMATIONS

The NC Math 2 Transformations Unit is a connection between the *Function* and *Geometry* domains and is described by NC.M2.F-IF.1 and NC.M2.F-IF.2 standards.

> ***NC.M2.F-IF.1*** *Extend the concept of a function to include geometric transformations in the plane by recognizing that: the domain and range of a transformation function f are sets of points in the plane; and the image of a transformation is a function of its pre-image.*

This standard requires students to extend the definition of *function:*

> *A relation that pairs every element in one set, called the domain, with exactly one element of a second set, called the range,*

by allowing the domain of a function to be a collection of ordered pairs. That is, instead of an input element that looks like ***x***, a transformation would have an input element that looks like **(*x*,*y*)**. Similarly, the range of a transformation is also a set of ordered pairs.

NC.M2.F-IF.2, is very explicit in extending the function notation (think **f(*x*,*y*)** instead of **f(*x*)**) to very specific transformation examples. This list of examples excludes many of the transformations that students can draw, because those transformations may be challenging to formalize with algebraic formulas.

> ***NC.M2.F-IF.2*** *Extend the use of function notation to express the image of a geometric figure in the plane resulting from a translation, rotation by multiples of 90 degrees about the origin, reflection across an axis, or dilation as a function of its pre-image.*

For example, the standard requires the use of function notation to express "rotation by multiples of 90 degrees about the origin". In general, rotations about the origin can be shown as the results of a transformation defined by multiplying a position by a 2x2 matrix. Also, it is possible to write function formulas that translate points through rotations of other degrees, but those generalizations are better done with polar coordinates. Both polar coordinates and matrix multiplication are beyond the scope of NC Math 2, thus this standard is very specific.

Mapping Notation Approach Examples	
$r_{x\text{-axis}}(x, y) = (x, -y)$	Reflection (in x - axis)
$T_{a,b}(x, y) = (x+a, y+b)$	Translation
$R_{0,90°}(x, y) = (y, -x)$	Counter-clockwise Rotation 90° about origin
$D_{0,k}(x, y) = (kx, ky)$	Dilation scale factor *k*

Along with the two Function standards in this unit, the remaining standards fall into the Geometry domain. NC.M2.G-CO.2-.6 call for students to experiment with rigid and non-rigid transformations and to verify relationships between pre-images and images through experimentation. This use of experimentation is consistent thru the remaining standards for the unit, though NC.M2.G-SRT attends more to dilation.

LOOKING AHEAD: NC MATH 2 – UNIT 4

"Writing a formal proof of a geometric result is the endpoint of a significant piece of mathematical investigation. It is not generally an activity to be undertaken on its own. In particular, it occurs *after* an invariance has been detected, conjectured, and tested against a context of variation; *after* an appropriate diagram has been constructed and understood; and *after* relevant definitions have been brought into play." *NCTM (2013)*

The standards call for a shift from the experimentation of Unit 1, into proving geometric theorems in Unit 4. The language of the standards and the structuring and ordering of Units 1 and 4 allow students time to explore, so that they can begin to conjecture about variance and invariance around geometric objects.

Looking at upcoming units, there exists a lot of overlap between Units 1 and 4, particularly with NC.M2.G-CO.6 and NC.M2.G-SRT.1. In Unit 4, students gain a more formal understanding of transformations and their importance for proving similarity and congruence (NC Math 2 MRI Document).

TOOLS OF THE TRADE

This intention of exploration and experimentation as a means of both discovery *and* justification enlarges the toolkit that students can use to determine geometric relationships. Formerly, the toolkit contained a ruler, a protractor, and a compass. Now students may also include mirrors for reflection, tracing paper for moving or folding, and geometry apps and software.

STUDENT'S THINKING ABOUT TRANSFORMATIONS

Considering the ways that students think about transformations as functions is an important part of both planning and making in the moment decisions about what students are mathematically thinking and doing. A key understanding of progressing students in conceptualizing transformations not as motions, but as functions is their understanding of domain. In a study of high school student's conceptions of transformations as functions, Hollebrands (2003) found that over the course of an instructional unit students held several levels of conceptions related to their understanding of domain. She found that as students progressed through the unit they progressed in conceptualizing domain as *only the labeled points on a pre-image*, to *all points on a pre-image*, and to *all points in the plane*.

QUESTIONS TO CONSIDER

- *What conceptions do your students commonly have about tranformations as functions?*
- *How could an attention to domain support them?*
- *How can you design appropriate interventions or tasks to advance or refine their conceptions?*

References

Hollebrands, K. F. (2003). High school students' understandings of geometric transformations in the context of a technological environment. *The Journal of Mathematical Behavior*, *22*(1), 55-72.

Sinclair, N., Pimm, D., & Skelin, M. (2012). *Developing essential understanding of geometry for teaching mathematics in grades 9-12*. National Council of Teachers of Mathematics.

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Join us as we journey together to support teachers and leaders in implementing mathematics instruction that meets needs of North Carolina students.

NC²ML MATHEMATICS ONLINE

For more information and resources please visit the NC DPI math wiki for instructions on accessing our Canvas page created in partnership with the North Carolina Department of Public Instruction by http://maccss.ncdpi.wikispaces.net/

North Carolina Collaborative for Mathematics Learning

www.nc2ml.org