WELCOME
The webinar will begin at 3:30. While you are waiting, please mute your sound. During the webinar please type all questions in the question/chat box in the go-to task pane on the right of your screen. As always, this webinar and the supporting materials will be available on our wikispace.
www.ncdpi.wikispaces.net

Making Mathematics Accessible
Department of Public Instruction Mathematics Consultants

The topic of the webinar today is Making Mathematics Accessible for all students. Over the next hour we will look at ways to make math meaningful and accessible for all students.

Mathematics Section Contact Information

Kitty Rutherford
Elementary Mathematics Consultant
919-807-3934
kitty.rutherford@dpi.nc.gov

Amy Scrinzi
Elementary Mathematics Consultant
919-807-3839
amy.scrinzie@dpi.nc.gov

Robin Barbour
Middle Grades Mathematics Consultant
919-807-3841
robin.barbour@dpi.nc.gov

Johannah Maynor
High School Mathematics Consultant
919-807-3842
johannah.maynor@dpi.nc.gov

Barbara Bissell
K-12 Mathematics Section Chief
919-807-3838
barbara.bissell@dpi.nc.gov

Susan Hart
Program Assistant
919-807-3846
susan.hart@dpi.nc.gov

During this webinar, we will answer questions on this topics presented. If you have questions that do not pertain to this webinar, please contact the math consultant for your grade band.
Please take a moment to read this poem.
These poignant words come from a student who describes a classroom as she sees it. We want students actively engaged in their learning – something which can only happen if classrooms become student-centered instead of teacher-centered.

The belief that ALL students can generate mathematical understandings must be in place before learning and growth can take place. In this sense ALL means ALL – the EC student, the English Language Learner, the struggling student, the average student and the AIG student – every child who enters your classrooms. Our efforts to make content meaningful and accessible must activate the tremendous potential that lies with all students.

The term “differentiation” is often used when thinking about addressing the needs of all learners. Carol Tomlinson, an author of many books on differentiation, contends: “Whenever a teacher reaches out to an individual or small group to vary his or her teaching in order to create the best learning experience possible, that teacher is differentiating instruction.”

Take a moment to reflect on ways you differentiate now. The challenge is to move one’s
focus from “ability” to “possibility” – shifting teachers’ attention away from activities that construct students as able or unable, directing attention instead toward strategies and situations that allow access for all students to learn mathematics. We must maintain a perspective that expects students to be successful when provided access to important ideas, and provide support as students make sense of these ideas.

Mathematics for Every Student Responding to Diversity, Grades 6 – 8 NCTM, pg. 10

One strategy used to make mathematics accessible to students with learning difficulties is the Concrete-Representational-Abstract instructional sequence, also referenced as CRA. This is a research-based strategy with the purpose of ensuring that students truly have a thorough understanding of the math concepts/skills they are learning. When students who have math learning problems are allowed to first develop a concrete understanding of the math concept/skill, then they are much more likely to perform that math skill and truly understand math concepts at the abstract level. This sequence helps to make the invisible visible through models, drawing, manipulatives, etc.

Additional information can be found at: http://www.coedu.usf.edu/main/departments/sped/mathvids/understanding/understanding.html#instruction
Here is an example of the Concrete – Representational – Abstract instructional sequence used to build understanding of the algorithm for division of fractions. Often students are presented problems such as these to help learn the algorithm before given problems in context.

How would you instruct students to divide 5 by 1/3?

PAUSE

Most people probably used the algorithm to invert and multiply to get an answer of 15. Correct answer but as a student I may be wondering why is it 15 when I am dividing – did not teachers tell me that dividing makes numbers smaller. And what does the 15 actually mean?

Utilizing the CRA instructional sequence, this problem can be modeled with 5 rectangles. The problem is asking for each of the 5 to be divided into thirds.
Division of Fractions

\[ 5 \div \frac{1}{3} = ? \]
\[ 5 \times 3 = 15 \]

Why?

Because there are three thirds in every box we have a total of 15 thirds.

It is important in the representational phase that students write down the math they are using to solve the problem – in this situation 5 times 3.

Continuing with the CRA sequence, students would be given more examples to complete until a pattern emerges and students realize that dividing by \( \frac{1}{3} \) is the same as multiplying by 3. This understanding can now be made more abstract using different numbers, noting that the process for division of fractions remains the same each time.
### Algorithms

**Algorithms without understanding**
- Errors practiced and hard to break
- Extensive practice time
- Limited retention

Think about the difference between just showing the solution with an algorithm and allowing students time to build the understanding. Deborah Lowenburg Ball in her research found that algorithms without understanding lead to:

- errors being practiced and hard to break
- extensive practice time
- limited retention

### Algorithms

**Algorithms with understanding**
- Conceptual development
- Reduction in practice time
- Extended retention and application

However, algorithms with understanding lead to:

- conceptual development
- reduction in practice time
- extended retention and application

In the CCSS using models to introduce concepts before algorithms is often mentioned. One such example is operations with decimals. Students in elementary grades are using concrete models for decimal operations. The standard for 6th grade requires students to fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.

Also note that this problem was introduced out of context. Imagine the richness in a task asks a student to divide 5 chocolate bars into thirds. Too often the context is introduced after the algorithm. Perhaps the context can lead to the algorithm and deeper mathematical understanding!
It is important that students of all ages and abilities explain the why-tos as well as the how-tos. For example, consider radian measurement. Can students explain why an angle measuring 180 degrees is equivalent to an angle measuring pi radians, or is the expectation only for students to convert from one form to the other? Explaining why gets to the deeper mathematical understanding for even the most advanced students.

Upper Level Option:
***For example, consider polynomial division. Are students expected to communicate or explain when and why using synthetic division provides more information than evaluating the polynomial at a particular value, or do they perform both algorithms without making these connections?
When planning,
“What task can I give that will build student understanding?”
rather than
“How can I explain clearly so they will understand?”
Grayson Wheatley, NCCTM, 2002
2/10/2012 • page 14

When developing algorithmic understanding through problem solving situations or tasks where students are expected to explain the “How-Tos”, we need to ask ourselves, ….

Share quote

Mathematical understanding is demonstrated when students can explain their reasoning, and problem solving in contextual situations requires this behavior. Therefore, we must provide every student access to rich mathematical experiences.

As we talk about the importance or accessibility, let’s look at a problem solving situation which should be accessible for all students.

Take a moment to read “The Border Problem” and identify anything problematic that may arise when students are faced with solving this problem.

Pause as participants identify and discuss.
Sue is tiling a 10 by 10 patio. She wants darker tiles around the border.

- How many tiles will she need for the border?
- Show the arithmetic you used to solve the problem.
- Describe your method.
- Explain why your method makes sense.
- Use algebraic expressions to write a rule for each method you found.

Initially, we could adjust the format of the problem by using bullets, which clearly identifies each question within the task students are expected to address. In addition, providing a visual gives clarity for the visual learner. These differentiation strategies could also serve as problem solving strategies in any instructional setting, especially when developing tests and quizzes.

Some students, especially ELL students, relate differently to the mathematics curriculum in the use of language. In the mathematics classroom, many terms are used in ways that differ from normal English usage. For example, the word “product” in mathematics refers to the result of multiplying two numbers, whereas in conventional English it describes something that has been completed.

So..., what words do you think would give students difficulty when solving this problem?

Pause for participants to identify some words.
Sue is tiling a 10 by 10 patio. She wants darker tiles around the border.

• How many tiles will she need for the border?
• Show the arithmetic you used to solve the problem.
• Describe your method.
• Explain why your method makes sense.
• Use algebraic expressions to write a rule for each method you found.

These are a few we identified, but you may have found more....

Sue Border
By Product
Show
Method
Sense
Rule
Expressions

Students may experience difficulty with the subtleties of the mathematics terminology. Mastery of this mathematical vocabulary should be taught not as an end in itself, but as a means of mastering more mathematics. This also applies to students who have not yet mastered basic mathematical skills, but who have the ability to problem solve.

As you can see, this task spans several grade levels and either part could be used as an opportunity for differentiation dependent on the students you are working with. For example, the last bullet in the task could be used with middle school or early high school students when formalizing their algebraic reasoning.

Now, it’s time for us to do some math!

Please work in groups to find methods of solving this problem. If the last bullet is a bit much, don’t worry, please address the others.

Pause for participants to solve task.

How many tiles are in the border of
Here are several possibilities or methods for showing your arithmetic but there are others. Another differentiation strategy is to present students with a task that creates scenarios that allow students to approach the same problem in different ways, resulting in the same solution.

In this scenario, the ultimate goal is to use the arithmetic expressions developed through exploring the structure and repeated reasoning to find an algebraic rule that models the situation.

Another expectation of students in this problem is describing and explaining their reasoning. This requires students to communicate orally or through written language. Many students have difficulty using language functions such as; discussing, explaining, writing, representing, reflecting, predicting, making inferences, and hypothesizing.

Keeping this in mind, let’s get back to the task at hand. The “Border” problem was posed to a class of 7th
and 8th grade students, where the teacher gave the students a writing assignment. They were to choose one of the methods presented for solving the problem, then she asked them to describe and explain their method so that someone who was not in class would understand their reasoning. She also told them to be sure to explain why their method works.

Please take two minutes to write a description of one of the methods you found and explain why the method works.

Pause

After the students completed their written assignment, the teacher brought the class together for a discussion on how to critique their reasoning and the reasoning of others while fulfilling the expectation of using precise mathematical language when communicating. After modeling the expected behaviors for this assignment, the teacher gave the students an opportunity to improve on their learning and this is an example of the result.

The work sample on the left occurred before the class collaborated and the one on the right was the second attempt.
Notice how the student demonstrates growth in explaining, representing, and presenting through the use of precise mathematical terminology.

Students may also have difficulty recording their work so we have provided an example on our wiki showing how this task can be modified to address this issue.

Take a minute to reflect back to when you were completing the task, what student behaviors were evident?

Pause

Working cooperatively provides the opportunity for students to use language related to the task while conversing, collaborating, and tutoring each another. By using authentic discourse, students have opportunities to refine their communication skills by negotiating meaning through talk.

Implementing the strategies suggested so far and developing student behaviors such as; discussing, explaining, writing, representing, and presenting are necessary for ALL students to experience success. Research indicates that active and regular participation in the classroom - not only reading and listening but also discussing, explaining, writing, representing, and presenting - is critical for success in mathematics.

Students understand best when they explore concepts through purposeful activities and
investigations. Through organizing key concepts and big ideas into rich tasks, teachers create learning experiences that give more time for students to become proficient and explore deeper mathematical concepts.

The behaviors we’ve identified, which are necessary for the success of all learners, align with the expectations of the CCSSM Practices which will be implemented and assessed next year.

These are the CCSS for Mathematical Practices

What relationships exist between the differentiation strategies and student behaviors discussed earlier and the Standards for Mathematical Practice?

Pause for participants to reflect!
• The suggested strategies and The Common Core State Standards for Mathematical Practice carry across all grade levels and are behaviors teachers should develop in all students.
• The appropriate use of mathematics terminology is not only an indicator of mathematical understanding but also indicates attention to mathematical precision.

The Common Core State Standards for Mathematical Practice are standards that define behaviors of
mathematically proficient students. The aspects of inquiry-based learning, such as discourse, questioning, investigating, observing, classifying and collecting and analyzing data creates environments that naturally lend themselves to the CCSSM Practices.

Kitty talks about Mathematical practices before sharing next slide:

Let’s talk about taking these extremely important practices into an elementary classroom. The Standards for Mathematical Practices are things mathematicians do to be proficient/good at mathematics, we, students, use these practices when solving problem and we don’t even realize we are using them. When I visited a classroom we focused on just 2 of the 8 standards:

• Making sense of problems and persevere in solving them.

4. Model with mathematics.

First we did a task, this was the mathematical task given to students.

After working collaboratively with a partner to solve the task, we, as a class discussed the two Mathematical Practices.

• Make sense of problems and persevere in solving them.

4. Model with mathematics.

Did you have to make sense of problem? What did you do to make sense the problem? Did you have
Then I asked them to think about the problem they just solved, our class discussion, illustrate how you used the practices to solving this task.

This is an example of one student’s illustration of mathematical practice 1. Making sense of problems and persevere in solving them.

*Read –slide*

This student explained to me he drew a picture of his friend with no hair because he not only did he have to persevered in mathematics but in everyday life with his is battle with cancer. Making real world connections to the practices

This is an example of another student’s illustration of mathematical practice 4. Model with mathematics.

*Read –slide*

This activity could be used as a strategy to assess student development of the mathematical practices over time.
Let’s take a minute and look at this high leverage elementary task……allow some time
Think about all the practices that would come into play as students work together to solve this.

This provides opportunity for conversation/interaction among students. This embraces the intent of the Standards for Mathematical Practices.

Now think about this problem… how does it compare the previous task/problem….. How are they different? Basic recall, low level of thinking. Students are not explaining their thinking about how and what they know.

Sometimes teachers give this type of assignment to students because it doesn’t require communication or reasoning. However, students should be provided the same opportunity as all students to learn content. Teachers should provide language support to enable ELLs to fully participate in high-demanding cognitive tasks/
When thinking about the concept of area and perimeter what mathematical terms come to mind?

In two minutes list all terms you can think of in the center box.

This is a strategy a teacher could use to active prior knowledge or assess what students currently know about a topic or concept. (any content area, such as science, social studies)

In their math notebook student could just draw this figure one large rectangle then a smaller one inside.

This could be what a student’s rectangle could look like after about two minutes.

After the task or as the unit develops students will add other mathematical terms to the outer square. At parent conferences this could be a tool used to share what student came knowing and how they have progressed over time. These terms would be used when student explain how they solved problems. The use of this vocabulary is essential in building strong foundational understanding in mathematical concepts.

When students are work together to solve problems sometimes all it takes is a gentle reminder: "What
words should I hear as I circulate around the room or as I read your math notebook?” “compose, decompose, etc…”

As noted in the Standards for Mathematical Practice all students are expected to be 6. **Attend to precision**, communicate **precisely** to others, give carefully formulated explanations to each other. This strategy will help students with this expectation.

Research shows that while frustration is viewed as a natural part of the learning process in countries such as Japan, we, here in the UAS often take steps to reduce the frustration of students. Problems which start out as high-level tasks can be reduced to low-level as teachers work to reduce frustration. It is important to **Allow** mathematics to be problematic for students.

- Individual differences can become a resource that can enhance instruction rather than a difficulty that hinders instruction as students share their own methods. It also helps students hear other methods of solving a problem and allows others to identify the advantages and disadvantages of the different methods presented.

- As the class discussion revolves around sharing, analyzing, and improving the methods, the spotlight shifts from people to ideas
- Mistakes become an opportunity to
Strategies for Developing Mathematical Understanding

3. Determine what mathematical information should be presented and when this information should be presented.
   • Presenting too much information too soon removed the problematic nature of problem
   • Presenting too little information can leave the students floundering

As discussed earlier in this webinar, we often make sure the student has all the tools to solve a problem rather than allowing the problem to produce the tools. Finding the balance is critical to mathematical understanding.

ENCOURAGING MATHEMATICAL DISCOURSE

Teachers:
   • Use effective questioning
   • Be nonjudgmental about a response or comment
   • Let students clarify their own thinking
   • Require several responses for the same question
   • Require students to ask a question when they need help.
   • Never carry a pencil

Finally, it is important to encourage mathematical discourse. When a student says “I don’t get it,” he may really be saying, “Show me an easy way to do this so I don’t have to think.” This is the reasoning behind requiring students to ask a question when they need help.

(Never carry a pencil) Too easy for the teacher to do the work for the student instead of asking thought-provoking questions to make the student think.

Source: Mathematical Teaching in the Middle School, April 2000, Never
Say Anything a Kid Can Say.

But Most Importantly…

Never Say Anything a Kid can Say!!

The more I talked, the less students were learning!

Please don’t let student sit like Cornstalks!

Our teachers come to class,
And they talk and they talk,
Til their faces are like peaches,
We don’t;
We just sit like cornstalks.
Cazden, 1976, p. 64

Revisit what a student wrote about their experience with teachers.

Are there any peaches or cornstalks in your school? Classroom?
All of our information is on this wiki space!

For additional information about the contents of this presentation, please contact your K-12 mathematics consultant.